

Matter sources for a Null Big Bang

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We consider the properties of stress-energy tensors compatible with a Null Big Bang, i.e., cosmological evolution starting from a Killing horizon rather than a singularity. For Kantowski-Sachs cosmologies, it is shown that if matter satisfies the Null Energy Condition (NEC), then (i) regular cosmological evolution can only start from a Killing horizon, (ii) matter is absent at the horizon, and (iii) matter can only appear in the cosmological region due to interaction with vacuum. The latter is understood phenomenologically as a fluid whose stress tensor is insensitive to boosts in a particular direction. We also argue that matter is absent in a static region beyond the horizon. All this generalizes the observations recently obtained for a mixture of dust and a vacuum fluid. If, however, we admit the existence of phantom matter, its certain special kinds (with the parameter $w \leq -3$) are consistent with a Null Big Bang without interaction with vacuum (or without vacuum fluid at all). Then in the static region there is matter with $w \geq -1/3$. Alternatively, the evolution can begin from a horizon in an infinitely remote past, leading to a scenario combining the features of a Null Big Bang and an emergent universe.

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I. INTRODUCTION

The remarkable discovery that our Universe is accelerating [1] and its explanation, in the framework of general relativity, in terms of the so-called dark energy have posed a number of questions. One of them concerns the possible interaction between dark energy and usual matter that satisfies the standard (weak, strong, null) energy conditions. The second one is a relationship between the algebraic structure of the stress-energy tensor of dark matter (which is believed to represent above 70 per cent of the modern energy density) and possible types of evolution. The third one is how these types of evolution depend on the possible interaction between the ingredients, and vice versa, how they select or restrict the possible kinds of sources.

The advent of this new and important source of gravity can also shed new light on such a long-standing problem of relativistic cosmology as that of the initial cosmological singularity. The latter is a state of the space-time geometry (and, most frequently, of matter as well) which cannot be described in the classical framework. The most common way of its understanding is to appeal to quantum gravity, assuming a quantum birth of the Universe, but there are a number of interesting attempts to avoid a singularity classically or semiclassically. Such attempts

can be classified as follows: (a) an eternal stationary or quasistationary state followed by expansion (the so-called “emergent universes”), (b) an indefinitely long contraction phase followed by a bounce or a number of bounces (e.g., nonsimultaneous bounces in different directions in an anisotropic Universe), (c) periodic or quasi-periodic evolution, and (d) cosmological expansion starting from a Killing horizon, with a static or stationary state in the absolute past.

Let us discuss the fourth opportunity, called a Null Big Bang (or simply a Null Bang (see [2, 3] and references therein), using a certain phenomenological description of dark energy. The properties of this phenomenon were recently discussed in Ref. [3] among other features of the class of regular homogeneous T-models with dustlike matter and a vacuum dark fluid. The latter is a variable generalization of the cosmological constant, able to account for the present acceleration of the Universe, see Eq. (3) below, and is distinguished by the property of invariance under boosts in a particular direction, related to symmetry of the model under study [4]. It was shown, in particular [3], in the framework of Kantowski-Sachs (KS) spherically symmetric cosmologies, that, for a mixture of dust and the vacuum dark fluid,

- (i) regular cosmological evolution can only begin with a Killing horizon,
- (ii) dust is absent at the horizon itself (and it was therefore concluded that it is also absent in the static region beyond the horizon) and
- (iii) dust can appear in the cosmological region only due

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to interaction with the vacuum fluid.

It thus follows that this kind of vacuum, whose origin may be related to quantum effects of matter fields [4, 5] can, in principle, address (though phenomenologically at this stage of the study) three important problems simultaneously: that of the nature of dark energy, that of the initial cosmological singularity and that of the origin of usual matter. It can be added that KS cosmologies are not excluded by modern observations if one assumes that their sufficiently early isotropization [6], and the latter may follow from the process of matter creation from vacuum — see a discussion and some estimates in [3].

The aim of the present paper is to show that all these three observations can be generalized to *any* kind of matter satisfying the null energy condition (NEC) — normal matter, for brevity. One of the motivations for such a study is that the equation of state of matter created from vacuum may strongly differ from that of dust. The choice of the KS geometry is natural due to its spherical symmetry and possible links to black hole physics. There are examples of such black hole configurations with phantom matter, which contain expanding universes beyond the horizon (“black universes” [7, 8]). A Null Big Bang can, in principle, also occur with other geometries, e.g., those with cylindrical or planar symmetries, which may be a subject of future studies.

It should be stressed that the geometric properties of Killing horizons (mostly in black hole physics) have been studied in much detail, see, e.g., the reviews [9, 10] and monographs [11, 12]. Strange as it may seem, some much simpler but physically important issues concerning the relationship between the properties of a cosmological horizon and matter which can support it, evaded attention. Our paper is trying to fill this gap.

The paper is organized as follows. In Section II we obtain analogues of the above items (i)–(iii) for any kind of normal matter. Section III briefly discusses the possible matter content of a static region in the past of a Null Big Bang (more details may be found in [13]). Section IV points out at possible horizons in an infinitely remote past, and Section V summarizes the results.

II. KANTOWSKI-SACHS COSMOLOGY

Consider a KS spherically symmetric cosmology with the metric

$$\begin{aligned} ds^2 &= b^2(t)dt^2 - a^2(t)dx^2 - r^2(t)d\Omega^2, \\ d\Omega^2 &= d\theta^2 + \sin^2\theta d\varphi^2, \end{aligned} \quad (1)$$

supported by a source with the stress-energy tensor

$$T_{\mu}^{\nu}(\text{tot}) = T_{\mu}^{\nu}(\text{vac}) + T_{\mu}^{\nu}(\text{matt}), \quad (2)$$

where

$$T_{\mu}^{\nu}(\text{vac}) = \text{diag}(\rho_v, \rho_v, -p_{v\perp}, -p_{v\perp}) \quad (3)$$

describes a vacuum fluid (defined by the condition $T_{0(\text{vac})}^0 = T_{1(\text{vac})}^1$, guaranteeing invariance of $T_{\mu}^{\nu}(\text{vac})$ under Lorentz boosts in the distinguished direction x [4]) and

$$T_{\mu}^{\nu}(\text{matt}) = \text{diag}(\rho_m, -p_{mx}, -p_{m\perp}, -p_{m\perp}) \quad (4)$$

is the contribution of matter, to be considered below in the most general form compatible with (1), as an anisotropic fluid.

In what follows, it is convenient to use the “quasiglobal” time coordinate, such that $b = a^{-1}$. The coordinate defined in this way, as well as its counterpart in static, spherically symmetric metrics, has very important properties [14, 15]: it always takes finite values $t = t_h$ at Killing horizons that separate static or cosmological regions of space-time from one another; furthermore, near a horizon, the increment $t - t_h$ is a multiple (with a nonzero constant factor) of the corresponding increments of manifestly well-behaved Kruskal-type null coordinates, used for analytic continuation of the metric across the horizon. This condition implies the analyticity requirement for both metric functions $a^2(t)$ and $r^2(t)$ at $t = t_h$. Though, for our consideration, it is quite sufficient to require that these functions belong to class C^2 of smoothness.

With this coordinate gauge, the combination of Einstein’s equations $\binom{0}{0} - \binom{1}{1}$ reads

$$\frac{2\ddot{r}}{r}a^2 = -8\pi(\rho_m + p_{mx}). \quad (5)$$

So, let us assume that there is a horizon at some $t = t_h$, such that, as $t \rightarrow t_h$, r is finite and

$$a^2(t) \approx a_0(t - t_h)^n, \quad n \in \mathbb{N}, \quad (6)$$

where n is the order of the horizon. Then, it immediately follows from (5) and the horizon regularity requirement (which implies analyticity of $r(t)$ and, in particular, finiteness of \ddot{r}) that

$$\rho_m + p_{mx} \rightarrow 0 \quad \text{as} \quad t \rightarrow t_h. \quad (7)$$

Now, we will assume $\rho_m \geq 0$ and consider different kinds of matter: the “normal” one that respects the NEC,

$$T_{\mu\nu}(\text{matt})\xi^{\mu}\xi^{\nu} \geq 0, \quad \xi_{\mu}\xi^{\mu} = 0, \quad (8)$$

and the “phantom” one that violates it. Taking in (8) the null vectors $\xi^{\mu} = (a, a^{-1}, 0, 0)$ and $\bar{\xi}^{\mu} = (a, 0, r^{-1}, 0)$, we obtain two necessary conditions for the validity of the NEC:

$$\rho_m + p_{mx} \geq 0, \quad (9)$$

$$\rho_m + p_{m\perp} \geq 0. \quad (10)$$

A. Normal matter

For normal matter, by definition, Eq. (9) holds, and consequently, according to Eq. (5), $\ddot{r} \leq 0$. So we can

repeat the argument of [3]: let the system be expanding ($\dot{r} > 0$) at some t_1 . Then, either $r \rightarrow 0$ at some earlier instant $t_s < t_1$ (which means a curvature singularity) or the singularity is not reached due to a Killing horizon at some instant $t_h > t_s$.

Thus *item (i) (see the Introduction) is valid not only for dust but for any normal matter.*

Let us assume that near the horizon the pressure of our matter behaves as

$$p_{mx} \approx w\rho_m, \quad w > -1. \quad (11)$$

(The case $w = -1$ is excluded since it is precisely a vacuum behaviour.) Then it immediately follows from (7) that *both* $\rho_m \rightarrow 0$ and $p_{mx} \rightarrow 0$ at the horizon, and the relation (11) refers to the first non-vanishing term of a Taylor expansion of the function $p_{mx}(\rho_m)$ near $\rho_m = 0$. This proves item (ii), namely, the statement that normal matter is absent at the horizon.

Note that both inferences, items (i) and (ii), have been obtained irrespectively of whether the normal matter obeys the conservation law $\nabla_\nu T_\mu^\nu(\text{matt}) = 0$ or interacts with a vacuum fluid. (The latter, by definition, does not contribute to the expression $T_0^0 - T_1^1$, relevant to the NEC.) They also do not depend on the behaviour of p_\perp and on the validity of the energy conditions other than the NEC.

Our next step is to show that the condition $\rho_m(t_h) = p_{mx}(t_h) = 0$ cannot take place for non-interacting normal matter, which will prove item (iii).

Assuming the absence of interaction between matter and vacuum, the conservation law $\nabla_\nu T_\mu^\nu = 0$ should hold for each of them separately. Taking the component with $\mu = 0$, we obtain

$$\dot{\rho}_m + \frac{\dot{a}}{a}(\rho_m + p_{mx}) + \frac{2\dot{r}}{r}(\rho_m + p_{m\perp}) = 0 \quad (12)$$

and

$$\dot{\rho}_v + \frac{2\dot{r}}{r}(\rho_v + p_{v\perp}) = 0. \quad (13)$$

As to the transverse pressure, we only assume that (at least in the limit $\rho \rightarrow 0$)

$$|p_\perp|/\rho < \infty. \quad (14)$$

It is a very weak restriction: indeed, for comparison, the dominant energy condition would require $|p_\perp|/\rho \leq 1$. Then the term in Eq. (12) with $2\dot{r}/r$, which is finite, can be neglected as compared with the term containing $\dot{a}/a \sim n/[2(t - t_h)] \rightarrow \infty$. Therefore, the leading order of the solution to (12) near the horizon reads

$$\rho_m = \text{const} \cdot a^{-(w+1)}, \quad (15)$$

which diverges as $a \rightarrow 0$ if $w > -1$, contrary to item (ii). Consequently, non-interacting normal matter cannot exist in a KS cosmology with a horizon.

Thus item (iii) has also been proved: normal matter could only appear after a Null Big Bang due to interaction with a sort of vacuum.

B. Phantom matter

Consider, for completeness, phantom matter with $w < -1$ in Eq. (11). Again, as with normal matter, the condition (7) implies that both ρ_m and p_{mx} vanish at the horizon (unless w is variable and tends to -1 at the horizon, which is a vacuumlike behaviour). According to (15), however, regular solutions to the conservation equation (12), with zero density and pressure at the horizon, do exist.

The NEC and other energy conditions are violated now. Moreover, Eq. (5) now leads to $\ddot{r} > 0$, so that expansion in r can begin from a nonsingular state with $r \neq 0$, or r can have a minimum, and the presence of a Killing horizon is not necessary for obtaining a nonsingular cosmology.

If there is a Killing horizon, further information on the system behaviour near the horizon can be obtained if, in addition to the conservation law, we take into account the Einstein equations, of which two independent components may be chosen as (5) and the $\binom{0}{0}$ equation that reads

$$\frac{1}{r^2}(1 + \dot{r}^2 a^2 + 2a\dot{a}\dot{r}) = 8\pi(\rho_m + \rho_v). \quad (16)$$

Near the horizon, assuming sufficient smoothness of the corresponding functions of t , we can write the Taylor expansions

$$\begin{aligned} a^2 &= a_n^2 (\Delta t)^n [1 + o(1)], \\ r(t) &= r_h + \Delta t \dot{r}_h + \frac{1}{2} \ddot{r}_h (\Delta t)^2 + o(\Delta t)^2, \\ \rho_m &= \rho_k (\Delta t)^k [1 + o(1)], \end{aligned} \quad (17)$$

where $\Delta t = t - t_h$ and the constants a_n , r_h , ρ_k are positive. Comparing Eqs. (17) and (15), we see that

$$k = -(w+1)\frac{n}{2} \Rightarrow w = -1 - \frac{2k}{n}. \quad (18)$$

It follows from Eq. (5) that $k \geq n$ where $k > n$ corresponds to $\ddot{r}_h = 0$. Thus

$$w \leq -3, \quad (19)$$

and, in the generic case $\ddot{r}_h \neq 0$, we have $w = -3$.

The remaining equation (16) gives in the main approximation (written for each term separately)

$$1 + na_n^2 (\Delta t)^{n-1} r_h \dot{r}_h = 8\pi r_h^2 [\rho_v(t_h) + \rho_k (\Delta t)^k]. \quad (20)$$

This leads to two opportunities:

(a) $\rho_v(t_h) \neq 0$. Then we may have

$$\begin{aligned} \text{either } n > 1 \text{ and } \rho_v(t_h) &= \frac{1}{8\pi r_h^2} \\ \text{or } n = 1 \text{ and } \rho_v(t_h) &= \frac{1}{8\pi r_h^2} (1 + a_1^2 r_h \dot{r}_h). \end{aligned} \quad (21)$$

- (b) $\rho_v(t_h) = 0$. Then, collecting zero-order terms, we obtain $n = 1$ (that is, the horizon is simple, Schwarzschild-like) and $\dot{r}_h = -1/(a_1^2 r_h)$ (i.e., a universe appearing in a Null Bang is initially contracting in the two spherical directions, $\dot{r} < 0$).

III. BEYOND THE HORIZON: A STATIC REGION

In this paper, we are dealing with essentially nonstatic geometries. Meanwhile, if there is a static region separated by a horizon from a KS region, one can deduce restrictions on the properties of matter in such a region in the same manner. This actually clarifies the conditions under which static, spherically symmetric black holes can exist inside matter distributions. Such an analysis has been performed in Ref. [13]. It turned out that a regular black hole can be in equilibrium with matter having

$$w_r = p_r/\rho = -n/(n+2k) \geq -1/3, \quad k \geq n,$$

where p_r is the radial pressure, n and k are positive integers and, as before, n is the order of the horizon. In the generic case $k = n$, this gives precisely the value $w = -1/3$ typical of a cloud of disordered cosmic strings (see [16] and references therein). Such a cloud, however, has an isotropic pressure, which is only a special case in our reasoning that uses the radial pressure only.

We see that, at different sides of a horizon, the kinds of admissible matter are different ($w \leq -3$ in the KS region and $w_r \geq -1/3$ in the static one). This looks natural since, as compared with the equation of state $p_{mx} = w\rho$, the roles of p_{mx} and ρ now interchange, which leads to the substitution $w \longleftrightarrow 1/w$.

Let us, however, recall that for any normal matter as well as for generic phantom matter we have proved the inferences (i)–(iii) formulated in the Introduction, and, in the absence of a fluid with $w_r = -1/3$, the static region preceding the cosmological evolution should be purely vacuum. Its properties should then coincide with those described in Refs. [2, 3]. In particular, this static region can be nonsingular; it then contains a regular centre with an asymptotically de Sitter geometry near it.

IV. A NULL BANG INFINITELY LONG AGO

As was already mentioned in the Introduction, there are several cosmological scenarios connected with attempts to avoid a singularity classically or semiclassically. In particular, there is a variant in which the Universe began its evolution in an infinite past from an almost static state with a nonzero scale factor, the so-called “emergent universes” [17], designated as (a) in the list of possibilities (see the Introduction).

We would like to point out here that, in the KS framework, there exists an intermediate variant of nonsingular evolution which combines the properties of variants (a)

and (d) (the latter is the main subject of this paper). We mean the situation that one of the scale factors in the metric (1), namely, $a(t)$, vanishes as $t \rightarrow -\infty$ while the other, $r(t)$, remains finite in the same limit, and both timelike and null geodesics starting from $t = -\infty$ are complete. This is what can be called a “remote horizon” in the past, by analogy with remote horizons in static space-times mentioned in [13, 15].

We will illustrate this opportunity with two examples of such a generic behaviour as $t \rightarrow -\infty$:

$$(A) \quad a \approx a_0 e^{Ht}, \quad r \approx r_0 + B e^{Ht}, \quad (22)$$

$$(B) \quad a \approx a_0 \left(\frac{t_0}{-t} \right)^q, \quad r \approx r_0 + r_1 \left(\frac{t_0}{-t} \right)^s, \quad (23)$$

where $a_0, r_0, r_1, H, t_0, s, q = \text{const} > 0$. Then, carrying out an analysis similar to that of Sec. II, we obtain in case (A) $w = -4$ and in case (B) $w = -3 - (s+2)/q < -3$. Eq. (16) then leads in both cases to the requirement $\rho_v \rightarrow 1/(8\pi r_0^2)$ as $t \rightarrow -\infty$. Also, in both cases, the conservation equation (13) leads to $p_{v\perp} \rightarrow 0$, i.e., the vacuum stress tensor should asymptotically have the structure

$$T_{\mu}^{\nu}(\text{vac}) = \text{diag}(\rho_v, \rho_v, 0, 0). \quad (24)$$

Indeed, Eq. (16) shows that the quantity $Z := 8\pi r^2 \rho_v - 1$ is, in case (A), at most of the order $O(e^{3Ht})$ and, in case (B), $Z = o(|t|^{-s-2})$. On the other hand, Eq. (13) may be rewritten in the form $2r\dot{r}p_{v\perp} = -\dot{Z}/8\pi$. Comparing the orders of magnitude at both sides of this equation, we obtain $p_{v\perp} \rightarrow 0$.

Thus a combination of the Null Big Bang and emergent universe scenarios is possible but only under some special conditions: $w \leq -3$ and a particular structure of the vacuum stress-energy tensor in the remote past.

V. DISCUSSION

We have considered KS cosmologies with a source representing a mixture of a vacuum dark fluid with the stress-energy tensor (3) and some non-vacuum matter. We have shown the following:

1. In the presence of normal matter, respecting the NEC, regular cosmological evolution can begin with a Killing horizon only. Assuming such regularity, hence the existence of a horizon, further properties are proven.
2. Normal matter is absent at the horizon. Items 1 and 2 are valid irrespective of whether or not normal matter obeys the conservation law, e.g., whether or not it interacts with the dark fluid.
3. Normal non-interacting matter cannot emerge in the cosmological region. It can only appear there

due to interaction with the dark fluid. This restriction is absent for phantom matter that violates the NEC, with some particular values of $w = p_{mx}/\rho \leq -3$, the value -3 being generic. Curiously, in the corresponding static region (if any), matter should have a non-phantom equation of state with $w \geq -1/3$, the value $-1/3$ being generic.

4. In the presence of phantom matter with $w \leq -3$, and the asymptotic (24) of the vacuum stress-energy tensor with $\rho_v \rightarrow \text{const} > 0$, the horizon may occur in an infinitely remote past, which leads to a scenario resembling that of an emergent universe.

Concerning configurations with normal matter, we can conclude that the static region, preceding a regular KS evolution, should be filled with a vacuum fluid only; the latter can provide the existence of a regular centre with an asymptotically de Sitter geometry [2, 3].

In our reasoning, relying on the asymptotic behaviour of the density and pressure near the horizon, we did not assume any particular equation of state and even did not restrict the behaviour of the transverse pressure except for its regularity requirement. In this sense, our conclusions are model-independent. The fact that the very assumption of the existence of a cosmological horizon entails a number of rather general conclusions resembles, to some extent, the situation in black hole physics where the presence of the horizon greatly simplifies the description of the system and reduces the number of possibilities.

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